3D Computer Vision

Selected Topics:

Photometric Stereo / Shape from Shading

Scanning from Shadows

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Outline

• Part 1: Photometric Stereo
  ▪ Radiometry
  ▪ Solid Angle
  ▪ Radiance and Irradiance
  ▪ BRDF
  ▪ Shape from Shading

• Part 2: Scanning from Shadows
  ▪ Scanning Principle and Setup Calibration
  ▪ Scanning with the Sun
Selected Topics - Part 1:

Photometric Stereo
What influences the brightness / intensity of an image pixel?
Radiometry - Radiance and Irradiance

- *Radiometry* investigates the behavior of the amount of light energy during image formation. Especially it deals with:
  - **Radiance (L):** Light energy emitted from light sources.
  - Light energy reflected from surfaces.
  - **Irradiance (E):** Light energy registered by sensors.
Radiometry - Solid Angle

• **Foreshortening**
  A big light source, viewed at a glancing angle, emits the same amount of light as a small source viewed frontally.

• **Arc angle**
  The scope of a circle is given by $2\pi r$. Recall that the arc length of a segment $dl$ is given by $dl = d\varphi \cdot r$.

• **Angle due to a tilted line segment**
  Using this information an effective angle can be computed from any tilted line segment:

  $$d\varphi = \frac{dl \cos \theta}{r}$$
Radiometry - Solid Angle

• Solid Angle

  - Two dimensional version of the effective angle.

  \[ d\phi = \frac{dl \cos \theta}{r} \implies d\omega = dA_0 = \frac{dA \cos \theta}{r^2} \]

  - Defined by the projected area of a surface patch \( dA \) onto a sphere of radius \( r \).
Radiometry - Radiance and Irradiance

- **Radiance (L)**
  - Energy carried by a ray.
  - Power per unit area perpendicular to the direction of travel (per unit solid angle). Unit: \( \frac{W_s}{m^2\cdot r} \)

- **Irradiance (E)**
  - Energy arriving at a surface.
  - Incident power per unit area not foreshortened
  - For a surface receiving radiance \( L \) coming from \( d\omega \) the corresponding irradiance is
    \[
    E = \frac{P}{dA} = \frac{LdA \cos(\theta) d\omega}{dA} = L \cos(\theta) d\omega
    \]
Radiometry - Image Intensity and Radiance

- **Radiometry of thin lenses**
  - $L$: Radiance emitted from $P$ toward $P'$.
  - $E$: Irradiance falling on $P'$ from the lens.

- **What is the relationship between $E$ and $L$?**
  \[
  E = \left[ \frac{\pi}{4} \left( \frac{d}{z'} \right)^2 \cos^4(\alpha) \right] L
  \]
  - Image irradiance is linearly related to scene radiance
  - Irradiance is proportional to the area of the lens and inversely proportional to the squared distance between the lens and the image plane.
  - The irradiance decreases with increasing angle between the viewing ray and the optical axis.
The mapping $f$ from irradiance to pixel values is called *camera response function*.

- Useful if we want to estimate material properties.
- Enables us to create high dynamic range images.
• What happens when a light ray hits a point on an object?
  ▪ Some of the light gets absorbed → converted to other forms of energy (e.g., heat)
  ▪ Some gets transmitted through the object → possibly bent, through “refraction” or scattered inside the object (subsurface scattering)
  ▪ Some gets reflected → possibly in multiple directions at once
  ▪ Really complicated things can happen → fluorescence

• Let’s consider the case of reflection in detail
  ▪ Light coming from a single direction could be reflected in all directions.
    How can we describe the amount of light reflected in each direction?
Radiometry
- Bidirectional Reflectance Distribution Function

• What is a BRDF?
  
  ▪ Model of local reflection that tells how bright a surface appears when viewed from one direction when light falls on it from another.
  
  ▪ Defined as ratio of the radiance in the outgoing direction to irradiance in the incident direction.
    
    \[ \rho(\theta_i, \phi_i, \theta_e, \phi_e) = \frac{L_e(\theta_e, \phi_e)}{E_i(\theta_i, \phi_i)} = \frac{L_e(\theta_e, \phi_e)}{L_i(\theta_i, \phi_i) \cos \theta_i d\omega} \]

  ▪ Radiance leaving a surface in a particular directions is the sum/integral of all contributions from every incoming direction:
    
    \[ \int_{\Omega} \rho(\theta_i, \phi_i, \theta_e, \phi_e) L_i(\theta_i, \phi_i) \cos \theta_i d\omega \]
Radiometry
- Bidirectional Reflectance Distribution Function

BRDF can be incredibly complicated.
Radiometry
- Bidirectional Reflectance Distribution Function

- **Diffuse reflection**
  - Light is reflected equally in all directions
  - Dull, matte surfaces like chalk or latex paint
  - Microfacets scatter incoming light randomly

- **Radiance of diffuse reflection is constant**
  - Brightness of the surface depends on the incidence of illumination.
  - Brightness of the surface does **not** depend on the viewing direction.
Radiometry
- Bidirectional Reflectance Distribution Function

- **Lambert’s Law for diffuse reflection**
  - **Radiosity** ($B$) is the total power leaving the surface per unit area (regardless of direction).
  - **Albedo** ($\rho$) is the fraction of incident irradiance reflected by the surface.
  - **Source vector** ($S$) is the magnitude proportional to intensity of the source.

\[
B = \rho N \cdot S = \rho \|S\|_2 \cos(\theta)
\]
Radiometry
- Bidirectional Reflectance Distribution Function

• **Specular reflection**
  - Radiation arriving along a source direction leaves along the specular direction (source direction reflected about normal)
  - Some fraction is absorbed, some reflected.
  - On real surfaces, energy usually goes into a lobe of directions.
Radiometry
- Bidirectional Reflectance Distribution Function

- Phong model for specular reflection
  - Reflected energy falls off with $\cos^n(d\theta)$
  - Lambertian + specular model:
    Sum of diffuse and specular term.

Moving the light source
Changing the exponent
Photometric Stereo - Shape from Shading

• Can we reconstruct the shape of an object based on shading cues?

→ Yes, it is called Photometric Stereo or Shape from Shading.

Luca della Robbia, *Cantoria*, 1438
• **Goal**
  - Reconstruct object shape and albedo from differently illuminated scene images.

• **Assumptions:**
  - A Lambertian object.
  - A local shading model (each point on a surface receives light only from sources visible at that point - we don’t consider other objects at all).
  - A set of known light source directions.
  - A set of pictures of an object, obtained in exactly the same camera/object configuration but using the different sources.
  - Orthographic projection.
Photometric Stereo - Image Model

- **Known**: Source vectors $S_j$ and image intensity values $I_j(x,y)$.
- **Unknown**: Surface normal $N(x,y)$ and albedo $\rho(x,y)$.

Assuming a linear response function of the camera: $I_j(x,y) = k \cdot B_j(x,y)$

And a diffuse surface fulfilling Lambert’s law: $B_j(x,y) = \rho(x,y)(N(x,y) \cdot S_j)$

$\Rightarrow I_j(x,y) = k\rho(x,y)(N(x,y) \cdot S_j) = (\rho(x,y)N(x,y)) \cdot (kS_j)$
• For every pixel $x, y$ we can setup a system of equations for $n$ different measured light sources $S_j$.

\[
\begin{pmatrix}
V_1^T \\
V_2^T \\
\vdots \\
V_n^T
\end{pmatrix}
\begin{pmatrix}
g(x, y)
\end{pmatrix}
= 
\begin{pmatrix}
I_1(x, y) \\
I_2(x, y) \\
\vdots \\
I_n(x, y)
\end{pmatrix}
\]

- We obtain a least-squares solution for $g(x, y)$.
- Since $N(x, y)$ is the unit normal we get by the magnitude of $g(x, y)$:
  \[
  \rho(x, y) = \|g(x, y)\|_2
  \]

- Then we receive the normal $N(x, y)$ by dividing the albedo:
  \[
  N(x, y) = \frac{g(x, y)}{\rho(x, y)}
  \]
Photometric Stereo - Synthetic Example

Recovered albedo

Recovered normal field
Photometric Stereo
- Recovering the Surface from Normals

- Investigating an arbitrary point on the surface \((x, y, s(x, y))\).

- Its normal \(N(x, y)\) has the form

\[
N(x, y) = \frac{1}{\sqrt{s_x^2 + s_y^2 + 1}} \begin{pmatrix} -s_x \\ -s_y \\ 1 \end{pmatrix}
\]

- And we moreover know

\[
N(x, y) = \frac{g(x, y)}{\rho(x, y)} = \frac{1}{\sqrt{g_1^2 + g_2^2 + g_3^2}} \begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix}
\]

- We therefore directly obtain the values for the partial derivatives \(s_x\) and \(s_y\):

\[
s_x(x, y) = -\frac{g_1(x, y)}{g_3(x, y)} \quad s_y(x, y) = -\frac{g_2(x, y)}{g_3(x, y)}
\]
Photometric Stereo
- Recovering the Surface from Normals

- For the surface $s$ to exist, the mixed second partial derivatives must be equal (Integrability). In practice, they should at least be similar:

$$\frac{\partial}{\partial y} \frac{g_1(x, y)}{g_3(x, y)} \approx \frac{\partial}{\partial x} \frac{g_2(x, y)}{g_3(x, y)}$$

- We can now recover the surface height at any point by integration along some path, e.g.

$$s(x, y) = \int_0^x s_x(t, y)\,dt + \int_0^y s_y(x, t)\,dt + c$$

- For robustness, can take integrals over many different paths and average the results.
Photometric Stereo - Synthetic Example

Recovered albedo
Recovered normal field
Recovered surface model
Photometric Stereo - Real Example

Recovered albedo

Recovered normal field

Recovered surface model
Photometric Stereo - Light Source Direction

The direction of a light source can be computed from an image in a similar way:

\[ I_j(x, y) = \frac{k \rho(x, y) N(x, y) \cdot S_j}{W(x, y)} \]

Every pixel of an image gives a measurement, leading again to a system of equations:

\[
\begin{pmatrix}
W_x(x_1, y_1) & W_y(x_1, y_1) & W_z(x_1, y_1) \\
W_x(x_2, y_2) & W_y(x_2, y_2) & W_z(x_2, y_2) \\
\vdots & \vdots & \vdots \\
W_x(x_n, y_n) & W_y(x_n, y_n) & W_z(x_n, y_n)
\end{pmatrix}
\begin{pmatrix}
S_x \\
S_y \\
S_z
\end{pmatrix}
= 
\begin{pmatrix}
I(x_1, y_1) \\
I(x_2, y_2) \\
\vdots \\
I(x_n, y_n)
\end{pmatrix}
\]

Photometric Stereo - Light Source Direction

Fake photos

Photometric Stereo - Limitations

- Orthographic camera model
- Simplistic reflectance and lighting model
- No shadows
- No inter-reflections
- No missing data
- Integration is tricky
- paths and average the results.
Selected Topics - Part 2:

Scanning from Shadows
Scanning from Shadows - Scanner Setup

Simple structured light setup for everybody.

Scanning from Shadows - Scanner Setup

Move stick over the scene.

Captured image with the moving shadow of the stick.

Scanning from Shadows - Scanning Principle

- **Geometrical principle**
  - The light source $S$ and the stick span a plane, where any encoded world point $P$ lies in.
  - World point $P$ is the intersection of the spanned plane $\Pi$ and the ray through the projected image point $p$.

\[
P = \overline{Op} \cap \Pi
\]
Scanning from Shadows - Scanning Principle

- Geometrical principle
  - We assume the object being on a planar desk $\Pi_d$. So plane $\Pi$ can be easily computed from the projection of the stick onto the desk $\Lambda$ and its projection in the captured image $\lambda$.

\[
\Lambda = O\lambda \cap \Pi_d \\
\Rightarrow \Pi = S\Lambda
\]
Scanning from Shadows - Scanning Principle

- **Geometrical principle**
  - Using an additional known plane in the background $\Pi_v$, we can even compute plane $\Pi$ without knowing the light source $S$.

\[
\begin{align*}
\Lambda_1 &= O\lambda_1 \cap \Pi_d \\
\Lambda_2 &= O\lambda_2 \cap \Pi_v \\
\Rightarrow \quad \Pi &= \Lambda_1 \Lambda_2
\end{align*}
\]
Scanning from Shadows - Scanner Calibration

- **Camera and desk plane calibration**
  - Place checkerboard on the desk
  - Calibrate the internal parameters of the camera
  - Calibrate the desk plane relative to the camera
    (see lecture on checkerboard calibration)
Scanning from Shadows - Scanner Calibration

• Lamp calibration
  - Place a pencil with known length $h$ on the desk.
  - Detect the edgepoints of the shadow $b$ and $t_S$ in the image.
  - Since we know that the pencil is placed orthogonal on the calibrated desk plane, we know world point $T$ from $B$ and $h$.
  - The direction, of light rays, emitted from the light source $S$ can easily be computed from $T$ and $T_S$.

$$S \in \Delta = \overrightarrow{TT_S}$$
Vertical plane calibration

- Detecting the edge $\lambda_I$ between desk plane and vertical plane is enough to compute $\Pi_v$ as the orthogonal plane to $\Pi_d$ containing $\lambda_I$.

\[
\Pi_v \perp \Pi_d \\
\lambda_I \in \Pi_v \implies \Pi_v
\]
Scanning from Shadows - Scanning

• **Spatio-temporal line detection**
  - Estimate the time, where the edge of the shadow passes through any pixel.
  - Estimate the pixel, where the edge of the shadow passes at a specific time.
  - Both processing tasks correspond to finding the edge of the shadow, but the search domains are different (spatial coordinates and temporal coordinate).
  - Combination has been shown to be appropriate for preserving sharp discontinuities in the scene.

Scanning from Shadows - Scanning Results

Scanning accuracy of 0.1mm over 10cm (~0.1% error).

Scanning from Shadows - Scanning with the Sun

Scanning accuracy of 1cm over 2m (~0.5% error).

Thank you!